$r - g$ Wealth trajectories and the Pareto tail

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Rates of return and Pareto tails

- A key claim of Piketty (2014) is that inequality rises if $r - g$ gets bigger,
- where
  1. r is the rate of return,
  2. g is the rate of growth of the economy.
- Where is this claim coming from?
- How does this relate to observed Pareto tails of the wealth distribution?

Wealth trajectories

- Consider the evolution of a given individual's wealth Y between time 0 and time 1.
- Wealth can grow because of
  1. Saved earnings w
  2. Reinvested returns rY plus principal, $R \cdot Y$
     Can think of R as $1 + s \cdot r$
     s is the savings rate
- Thus:

$$Y_1 = w + R \cdot Y_0$$

- If wealth is measured relative to aggregate income, subtract g from R.

Random returns

- Suppose now further that returns are random:

$$R \perp Y_0$$

- And suppose that w is negligible for the really wealthy people.
- For these, therefore,

$$Y_1 \approx R \cdot Y_0$$

Stationary distribution

- People randomly move up and down the wealth distribution.
- When these movements net out in aggregate, we have a "stationary distribution."
- Formally:

$$P(Y_0 > y | Y_0 \geq \underline{y}) = P(Y_1 > y | Y_1 \geq \underline{y})$$

- One can show that the distribution of Y converges to a stationary Pareto distribution if $E[R] < 1$.
- Thus:

$$P(Y_0 > y | Y_0 \geq \underline{y}) \approx \left(\underline{y}/y\right)^{\alpha_0}.$$

How is α_0 determined?

- What Pareto parameter yields a stationary tail?
- Given our assumptions,

$$\begin{aligned} P(Y_0 > y) &= P(Y_1 > y) \\ &= P(w + R \cdot Y_0 > y) \\ &\approx P(Y_0 > y/R) \\ &= E[P(Y_0 > y/R|R)]. \end{aligned}$$

- This follows from
  1. stationarity,
  2. the equation relating Y_1 to Y_0,
  3. and the law of iterated expectations.

- Plugging the Pareto distribution for Y_0 into these expressions we get

$$\left(\underline{y}/y\right)^{\alpha_0} = E\left[\left(\underline{y}/(y/R)\right)^{\alpha_0}\right].$$

- Divide by $\left(\underline{y}/y\right)^{\alpha_0}$ on both sides $\Rightarrow$

$$E\left[R^{\alpha_0}\right] = 1.$$

- We have derived the equation mapping the distribution of R to the Pareto parameter α_0.

Intuition

- Rich individuals move up the wealth distribution if their $R > 1$, they move down if their $R < 1$.
- Stationarity requires that upward and downward movements cancel each other – as many individuals move down as up.
- If it's more likely to move down than up (R is mostly small), then there are fewer people that are very rich rather than just rich.
  $\Rightarrow$ large α, little inequality
- If it's equally likely to move up as down (R is centered close to 1), there are almost as many very rich people as rich people.
  $\Rightarrow$ small α, lots of inequality