Empirical Research on Economic Inequality
Intergenerational mobility and measurement error

Maximilian Kasy

Harvard University, fall 2015

Equality of opportunity

- To what extent is equality of opportunity a reality?
- Has it changed over time? Does it differ across countries?
- Often translated as:
  To what extent does family background determine life chances, and, in particular, income?
- The question is less well defined than it might seem.
- There are several alternative objects one might try to estimate.

Object 1

- **Predictability** of (log) child income in a given year s (or a few years) **using** (log) parent **income in a given year** t (or a few years):

$$E[Y_{c,s}|Y_{p,t}]$$

- Expressed as elasticity (regression slope):

$$\frac{\text{Cov}(Y_{p,t}, Y_{c,s})}{\text{Var}(Y_{p,t})}$$

- If $Y = \log$ income:
  Percentage increase in an average child's income for a 1% increase in parent income
- Most common measure of intergenerational mobility

Object 2

- **Predictability** of (log) child's lifetime income **using** (log) parent's **lifetime income**:

$$E[\overline{Y}_c|\overline{Y}_p]$$

- Expressed as elasticity (regression slope):

$$\frac{\text{Cov}(\overline{Y}_p, \overline{Y}_c)}{\text{Var}(\overline{Y}_p)}$$

- Life cycle of earnings, transitory shocks, measurement error
  $\Rightarrow$ Income in given year varies a lot around lifetime income.
  $\Rightarrow$ Lifetime income is in general more strongly related between parents and children (more later).
- Lifetime income usually not available in data

Object 3

- **Predictability using additional variables:**

$$E[\overline{Y}_c | \overline{Y}_p, X_p, W_p]$$

- Expressed as elasticities (regression slopes):

$$\mathrm{Var}((\overline{Y}_p, X_p, W_p))^{-1} \cdot \mathrm{Cov}((\overline{Y}_p, X_p, W_p), \overline{Y}_c).$$

- Motivation: Why stop at parental income?
  Other factors such as parent education, location of residence, etc., also predict a child's outcomes and are "morally arbitrary."
- The more predictive factors we consider, the better we can predict a child's outcomes.

Object 4

- The **causal effect of** parent **lifetime income**:

$$\overline{Y}_c = g(\overline{Y}_p, \varepsilon).$$

- Not all correlations are causal – do we care about prediction or causality?
- Example: Parent and child incomes might be correlated because parental education has a causal effect, but not parental income.
- Notation: If parent income is changed, g and ε do *not* change, describing counterfactual (cf. potential outcomes)

Object 5

- The **causal effect of additional variables**:

$$\overline{Y}_c = h(\overline{Y}_p, X_p, W_p, \varepsilon')$$

- Combines 3 and 4.

Current and lifetime income

- Suppose we are interested in prediction using parental lifetime income,

$$\beta = \frac{\text{Cov}(\overline{Y}_p, \overline{Y}_c)}{\text{Var}(\overline{Y}_p)}$$

but only have prediction using current parental income,

$$\gamma = \frac{\text{Cov}(Y_{p,t}, Y_{c,s})}{\text{Var}(Y_{p,t})}$$

- How are the two related?

Classical measurement error

- Suppose that

$$Y_{p,t} = \overline{Y}_p + \varepsilon_{p,t}$$
$$Y_{c,s} = \overline{Y}_c + \varepsilon_{c,s},$$

- where

$$\mathrm{Cov}(\overline{Y}_p, \varepsilon_{p,t}) = \mathrm{Cov}(\overline{Y}_p, \varepsilon_{c,s}) = 0$$
$$\mathrm{Cov}(\overline{Y}_c, \varepsilon_{c,s}) = \mathrm{Cov}(\overline{Y}_c, \varepsilon_{p,t}) = 0$$
$$\mathrm{Cov}(\varepsilon_{p,t}, \varepsilon_{c,s}) = 0.$$

Questions for you

Interpret these equations.

Questions for you

- Recall
$$\gamma = \frac{\text{Cov}(Y_{p,t}, Y_{c,s})}{\text{Var}(Y_{p,t})}.$$
- Express the numerator and denominator in terms of covariances involving $Y_{p,t}$, $Y_{c,s}$.
- How does the result relate to
$$\beta = \frac{\text{Cov}(\overline{Y}_p, \overline{Y}_c)}{\text{Var}(\overline{Y}_p)}?$$

Solution

- Numerator:

$$\begin{aligned}
&\mathrm{Cov}(Y_{p,t}, Y_{c,s}) \\
&= \mathrm{Cov}(\overline{Y}_p, \overline{Y}_c) + \mathrm{Cov}(\overline{Y}_p, \varepsilon_{c,s}) + \mathrm{Cov}(\varepsilon_{p,t}, \overline{Y}_c) + \mathrm{Cov}(\varepsilon_{p,t}, \varepsilon_{c,s}) \\
&= \mathrm{Cov}(\overline{Y}_p, \overline{Y}_c).
\end{aligned}$$

- Denominator:

$$\mathrm{Var}(Y_{p,t}) = \mathrm{Var}(\overline{Y}_p) + 2 \cdot \mathrm{Cov}(\varepsilon_{p,t}, \overline{Y}_p) + \mathrm{Var}(\varepsilon_{p,t}) = \mathrm{Var}(\overline{Y}_p) + \mathrm{Var}(\varepsilon_{p,t}).$$

- Therefore,

$$\gamma = \frac{\mathrm{Var}(\overline{Y}_p)}{\mathrm{Var}(\overline{Y}_p) + \mathrm{Var}(\varepsilon_{p,t})} \cdot \beta.$$

Interpretation

- The short run coefficient γ is smaller than the long run coefficient β.
- This is called attenuation bias.
- The error on the left hand side ($\varepsilon_{c,s}$) does not matter.
- Attenuation bias is larger, the larger $\text{Var}(\varepsilon_{p,t})$.

Non-classical measurement error

- So far we have assumed errors ε are random ("classical").
- But there might be systematic differences between $\overline{Y}_c$ and $Y_{c,t}$ – "non-classical" measurement error.
- For instance, lifetime profile of earnings:

$$Y_{c,s} = \overline{Y}_c \cdot (1 + \alpha \cdot (s - \overline{s})) + \varepsilon_{c,s}.$$

Questions for you

Interpret this equation.

Questions for you

Assuming

$$Y_{c,s} = \overline{Y}_c \cdot (1 + \alpha \cdot (s - \overline{s})) + \varepsilon_{c,s}$$
$$Y_{p,t} = \overline{Y}_p + \varepsilon_{p,t},$$

calculate

$$\gamma = \frac{\text{Cov}(Y_{p,t}, Y_{c,s})}{\text{Var}(Y_{p,t})}.$$

Solution

$$
\begin{aligned}
& \text{Cov}(Y_{p,t}, Y_{c,s}) \\
&= \text{Cov}(\overline{Y}_p, \overline{Y}_c \cdot (1 + \alpha \cdot (s - \overline{s}))) + \text{Cov}(\overline{Y}_p, \varepsilon_{c,s}) \\
&\quad + \text{Cov}(\varepsilon_{p,t}, \overline{Y}_c \cdot (1 + \alpha \cdot (s - \overline{s}))) + \text{Cov}(\varepsilon_{p,t}, \varepsilon_{c,s}) \\
&= (1 + \alpha \cdot (s - \overline{s})) \cdot \text{Cov}(\overline{Y}_p, \overline{Y}_c),
\end{aligned}
$$

so that

$$
\gamma = (1 + \alpha \cdot (s - \overline{s})) \cdot \frac{\text{Var}(\overline{Y}_p)}{\text{Var}(\overline{Y}_p) + \text{Var}(\varepsilon_{p,t})} \cdot \beta .
$$

Questions for you

What happens if $s < \overline{s}$?

Practical relevance

- Given data constraints, child's earnings often observed at young age.
- Earnings profiles are steeper, but start later, for those with higher education.
- Underestimate inequality and overestimate mobility if $s < \overline{s}$.
- Summarizing
  1. Classical measurement error for parents leads to overestimated inequality $\mathrm{Var}(\overline{Y}_p) + \mathrm{Var}(\varepsilon_{p,t})$, attenuation bias
  2. Non-classical measurement error for children leads to underestimated inequality, bias factor $(1 + \alpha \cdot (s - \overline{s}))$

Remedies

1. Use better data.
   e.g. IRS data in Chetty et al. (2014), rather than self-reported earnings which have more measurement error.
2. Average earnings over several years.

Average earnings

- Earnings of k years:

$$\frac{1}{k}\sum_{t=t_0}^{t_0+k} Y_{p,t} = \overline{Y}_p + \frac{1}{k}\sum_{t=t_0}^{t_0+k} \varepsilon_{p,t}$$

- Variance of error:

$$\text{Var}\left(\frac{1}{k}\sum_{t=t_0}^{t_0+k} \varepsilon_{p,t}\right) = \frac{1}{k^2}\sum_{t=t_0}^{t_0+k} \text{Var}(\varepsilon_{p,t}) = \frac{1}{k}\text{Var}(\varepsilon_{p,t_0}).$$

- Attenuation bias factor:

$$\frac{1}{1+\frac{1}{k}\frac{\text{Var}(\varepsilon_{p,t})}{\text{Var}(\overline{Y}_p)}}.$$

Remedies continued

3. Assess the reliability of the data

- Can get a sense of the amount of bias using repeated measurements
- "Reliability ratio":

$$\mathrm{Corr}(Y_{p,t_1}, Y_{p,t_2}) = \frac{\mathrm{Cov}(Y_{p,t_1}, Y_{p,t_2})}{\sqrt{\mathrm{Var}(Y_{p,t_1}) \cdot \mathrm{Var}(Y_{p,t_2})}} = \frac{\mathrm{Var}(\overline{Y}_p)}{\mathrm{Var}(\overline{Y}_p) + \mathrm{Var}(\varepsilon_{p,t})}.$$

- Same as formula for attenuation bias!

Remedies for non-classical measurement error

4. Measure child's income later in life, so that

$$(1 + \alpha \cdot (s - \bar{s})) \approx 1.$$

5. Consider other outcomes which are determined earlier, e.g., education.

References

Chetty, R., Hendren, N., Kline, P., and Saez, E. (2014). Where is the land of opportunity? The geography of intergenerational mobility in the United States. Quarterly Journal of Economics*, 129(4):1553–1623.*

Lee, C. and Solon, G. (2009). Trends in intergenerational income mobility. The Review of Economics and Statistics*, 91(November):766–772.*

Black, S. and Devereux, P. (2011). Recent developments in intergenerational mobility. Handbook of Labor Economics*, 4:1487–1541.*